EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 27, 155–171 (1998)

EQUIPMENT–STRUCTURE INTERACTION CONSIDERING THE EFFECT OF TORSION AND BASE ISOLATION

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SUMMARY

This paper attempts to study the response of equipment items attached to torsional buildings supported by elastic bearings under earthquake excitations. To account for the effect of torsion and translation, each storey of the building is modelled with two degrees of freedom, one for translation and the other for torsion. The equipment is assumed to be so light that it affects slightly the vibration modes of the primary structure to which it is attached. Modal synthesis results obtained by the perturbation technique together with the CQC procedure are compared with those from a complete eigenvalue analysis. Using the present semi-analytical approach, the key parameters that govern the equipment and structure responses can be easily identified. In the numerical studies, it is confirmed that the response of the equipment and the building to which it is mounted, can be effectively reduced through installation of the base isolators. The optimal point for mounting the equipment is the one where the equipment remains undisplaced during vibration of the tuned mode. © 1998 John Wiley & Sons, Ltd.

Earthquake Engng. Struct. Dyn., **27**, 155–171 (1998)

KEY WORDS: equipment; equipment–structure interaction; isolation; perturbation method; torsional building

INTRODUCTION

Previous researches on the aseismic response of equipment items mounted on structures have been directed generally toward investigation of the effects of tuning, interaction, non-classical damping, and spatial coupling. The works that have been conducted along this line include those of Kelly and Sackman,[1] Sackman *et al.*,[2] Igusa and Der Kiureghian,[3] Suarez and Singh,[4] among others. In addition to the aforementioned effects, the effect of base isolation on the equipment–structure systems has been studied by Kelly and Tsai.[5,6] A review of the existing literature, however, indicated that little attention has been directed toward the effect of torsion of the primary structure to which the equipment is mounted, and the effect of eccentricity of the equipment. This paper can be regarded as an extension of the authors' earlier work[7] on the response of equipment items mounted on torsional buildings to include the effect of base isolation. The isolation devices considered herein are those made of the laminated rubber bearings, which have very low stiffness against the horizontal loads, but high stiffness against vertical loads.

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Contract grant sponsor: National Science Council of the Republic of China; Contract grant numbers: NSC83-0416-E002-018, NSC84-2221-E002-049

CCC 0098–8847/98/020155–17$17·50

Received 17 September 1996
Revised 18 March 1997

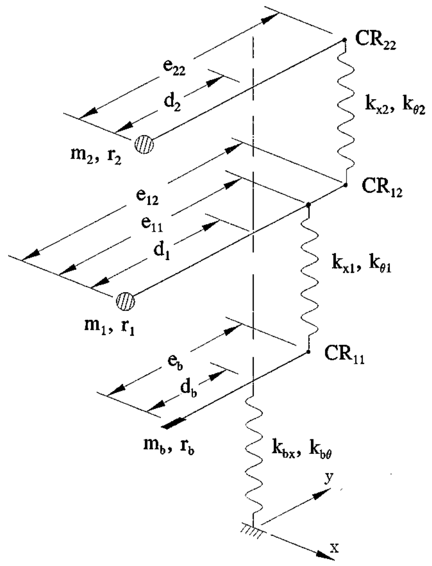

Figure 1. Model for base-isolated torsional building

One basic assumption for the torsional building is that the centre of mass (CM) of each floor of the building is offset from its centre of rigidity (CR) in the same direction perpendicular to the seismic input force. With this model, one need only concentrate on the torsion and translation of the system in one direction, thereby reducing the algebraic complexity involved, while still illustrating the effect of concern satisfactorily. By the method of perturbation, approximate closed-form expressions will be obtained for the modal properties of the combined system in terms of those of the subsystems. The damping property of the system is considered through the spectrum and the coefficient of correlation in modal synthesis. The present semi-analytical solutions are compared with those obtained by solving the complete eigenvalue problem for the combined system. Unlike the procedures that rely fully on numerical computations, the present approach has the advantage that it can help identify the key parameters governing the equipment and structure responses.

MATHEMATICAL MODEL

Consider a torsional building, the CM of each floor of which is offset from its CR along the y-axis. In Figure 1, e_{ij} denotes the y-direction eccentricity of the CR of the jth floor with respect to the CM of the ith floor. The other symbols used in Figure 1 are defined as follows: m = mass, k = stiffness, r = radius of gyration, and d = distance of the CM of each floor relative to the CR of the isolated base. Throughout this study, the following definitions are adopted for the subscripts attached to each quantity: i = ith floor, t = total system, s = superstructure, b = base; x = x-direction, and θ = rotation. A matrix will be denoted within a pair of brackets [ ] and a vector within braces { }.

Based on the rigid floor assumption, each storey of the building is modelled with two degrees of freedom (DOFs), namely, the translation u of the CM and the rotation θ about the vertical axis. By considering the superstructure and the base as a free body, the equations of motion can be written for the two directions as

$$\begin{aligned} & m_t \ddot{u}_b + \left(\sum m_i d_i + m_b d_b\right)\ddot{\theta}_b + ([m]_s\{l_x\}_s)^T\{\ddot{u}\}_s + k_{bx}u_b = -\, m_t \ddot{u}_g \\ & \left(\sum m_i d_i + m_b d_b\right)\ddot{u}_b + m_t r_t^2 \ddot{\theta}_b + ([m]_s\{l_\theta\}_s)^T\{\ddot{u}\}_s + k_{b\theta}\theta_b = \left(\sum m_i d_i + m_b d_b\right)\ddot{u}_g \end{aligned} \quad (1)$$

where $m_t = \Sigma m_i + m_b$, $r_t^2 = [\Sigma m_i(r_i^2 + d_i^2) + m_b(r_b^2 + e_b^2)]/m_t$, u_b and θ_b denote, respectively, the displacement and rotation of the CR of the base relative to the ground, $[m]_s$ is the mass matrix of the superstructure, $\{u\}_s$ the displacements of the superstructure relative to the CR of the base, and $\{l_x\}_s$ and $\{l_\theta\}_s$ denote, respectively, the displacement and rotation influence vector of the superstructure, i.e., $\{l_x\}_s = \{1\,0\,1\,0\,\ldots\,1\,0\}^T$ and $\{l_\theta\}_s = \{d_1\,1\,d_2\,1\,\ldots\,d_n\,1\}^T$. On the other hand, by considering each storey of the building as a free body, two equations of motion can be written for the two directions. The following are the equations of motion for the building that contains n storeys:

$$[m]_s\{\ddot{u}\}_s + [m]_s\{l_x\}_s\ddot{u}_b + [m]_s\{l_\theta\}_s\ddot{\theta}_b + [k]_s\{u\}_s = -[m]_s\{l_x\}_s\ddot{u}_g \tag{2}$$

where $[k]_s$ denotes the stiffness matrix of the superstructure.

The foregoing three equations of motion can be combined and written as

$$[m]\{\ddot{u}\} + [k]\{u\} = -[m]\{l_x\}\ddot{u}_g \tag{3}$$

where $[m]$ and $[k]$ denote the mass and stiffness matrices of the entire system, $\{u\}$ the displacements relative to the ground, and $\{l_x\}$ the displacement influence vector,

$$[m] = \begin{bmatrix} m_t & \sum m_i d_i + m_b d_b & ([m]_s\{l_x\}_s)^T \\ \sum m_i d_i + m_b d_b & m_t r_t^2 & ([m]_s\{l_\theta\}_s)^T \\ ([m]_s\{l_x\}_s)^T & ([m]_s\{l_\theta\}_s)^T & [m]_s \end{bmatrix}$$

$$[k] = \begin{bmatrix} k_{bx} & 0 & 0 \\ 0 & k_{b\theta} & 0 \\ 0 & 0 & [k]_s \end{bmatrix}, \qquad \{u\} = \begin{Bmatrix} u_b \\ \theta_b \\ \{u\}_s \end{Bmatrix}, \qquad \{l_x\} = \begin{Bmatrix} 1 \\ 0 \\ \{0\} \end{Bmatrix} \tag{4a–d}$$

And the displacement $\{u\}_{sg}$ of the superstructure relative to the ground is

$$\{u\}_{sg} = \{u\}_s + \{l_x\}_s u_b + \{l_\theta\}_s \theta_b \tag{5}$$

in which θ_b represents the effect of torsion of the base isolators.

EFFECT OF BASE ISOLATION ON EQUIPMENT RESPONSE

In this section, the deformation of the superstructure will be approximated by a vibration mode in tuning with the equipment. Through comparison of the response derived for an equipment item mounted either on a structure with fixed base or supported by elastic bearings, the effect of base isolation on the equipment response will be evaluated. The present procedure is semi-analytical in nature, which differs from that of Kelly and Tsai[5] in that the effect of torsion of the building is taken into account.

Response of equipment mounted in torsional building with fixed base

Consider the case where a light equipment item is eccentrically attached to the jth floor of an n-storey torsional building with fixed base. Referring to Figure 2, the following quantities are defined for the equipment: m_e = mass, k_e = spring constant, u_e = translation and ϵ = eccentricity of equipment from the CM of the jth floor. The equations of motion for the combined equipment–structure system can be obtained

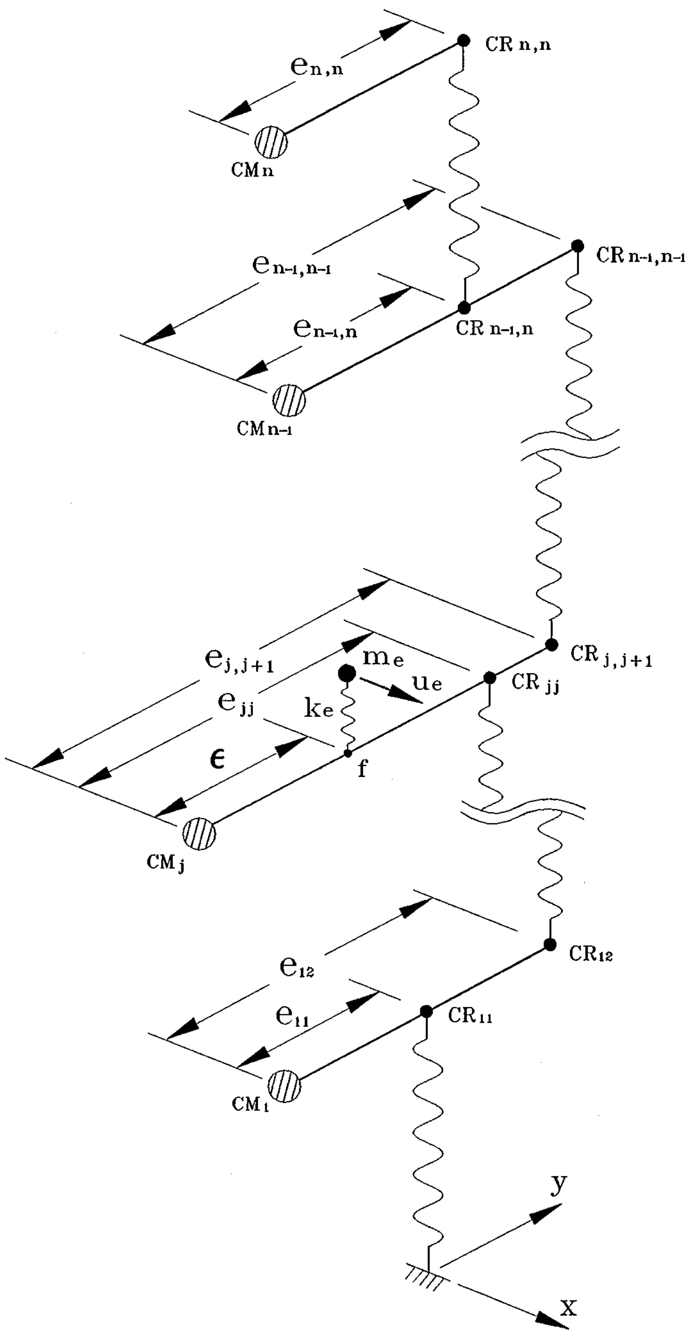

Figure 2. Model for equipment–structure system with fixed base

through modification of equation (2) as follows:

$$\begin{gathered}[m]_s\{\ddot{u}\}_s + [k]_s\{u\}_s - k_e(u_e - u_j + \varepsilon\theta_j)\{e\}_{2j-1} + k_e\varepsilon(u_e - u_j + \varepsilon\theta_j)\{e\}_{2j} = -[m]_s\{l_x\}_s\ddot{u}_g \\ m_e\ddot{u}_e + k_e(u_e - u_j + \varepsilon\theta_j) = -m_e\ddot{u}_g\end{gathered} \tag{6}$$

where $\{e\}_k$ is a unit vector of dimension $2n$ with its kth component equal to one and the remaining equal to zero.

Let ω_i denote the ith modal frequency and $\{\phi\}_i$ the associated modal shape of the structure with fixed base. The modal mass M_i and modal participation factor Γ_{xi} can be defined as

$$M_i = \{\phi\}_i^{\mathrm{T}}[m]_{\mathrm{s}}\{\phi\}_i, \quad \Gamma_{xi} = \{\phi\}_i^{\mathrm{T}}[m]_{\mathrm{s}}\{l_x\}_{\mathrm{s}}/M_i \tag{7a,b}$$

For the purpose of calculating the equipment response, let us transform the displacement and rotation of the jth floor from the CM to the attachment point f of the equipment,

$$\begin{Bmatrix} u_j \\ \theta_j \end{Bmatrix} = \begin{bmatrix} 1 & \varepsilon \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_f \\ \theta_f \end{Bmatrix} \tag{8}$$

while keeping the other DOFs unchanged. By this relation, the equations of motion (6) can be transformed to

$$\begin{aligned} &[m]_{\mathrm{s}}^*\{\ddot{u}\}_{\mathrm{s}}^* + [k]_{\mathrm{s}}^*\{u\}_{\mathrm{s}}^* - k_e(u_e - u_f)\{e\}_{2j-1} = -[m]_{\mathrm{s}}^*\{l_x\}_{\mathrm{s}}^*\ddot{u}_{\mathrm{g}} \\ &m_e\ddot{u}_e + k_e(u_e - u_f) = -m_e\ddot{u}_{\mathrm{g}} \end{aligned} \tag{9a,b}$$

where each quantity of the transformed system has been starred. The transformation does not affect the modal frequency ω_i, but the modal vector $\{\phi\}_i^*$ of the system:

$$(\{\phi\}_i^*)^{\mathrm{T}} = \{\phi_1\ \phi_2\ \cdots\ (\phi_{2j-1} - \varepsilon\phi_{2j})\ \phi_{2j}\ \cdots\ \phi_{2n}\}_i \tag{10}$$

By letting the $(2j-1)$th component, i.e. the translation of the attachment point of the equipment, equal unity, the modal vector $\{\phi\}_i^*$ can be normalized:

$$(\{\phi\}_i^*)^{\mathrm{T}} = \{\phi_1^*\ \ \phi_2^*\ \cdots\ 1\ \ \phi_{2j}^*\ \cdots\ \phi_{2n}\}_i \tag{11}$$

Correspondingly, the ith modal mass M_i^* and modal participation factor Γ_{xi}^* for the transformed system can be calculated as follows:

$$\begin{aligned} M_i^* &= (\{\phi\}_i^*)^{\mathrm{T}}[m]_{\mathrm{s}}^*\{\phi\}_i^* = \frac{M_i}{(\phi_{2j-1,i} - \varepsilon\phi_{2j,i})^2} \\ \Gamma_{xi}^* &= (\{\phi\}_i^{*\mathrm{T}}[m]_{\mathrm{s}}^*\{l_x\}_{\mathrm{s}}^*/M_i^* = \Gamma_{xi}(\phi_{2j-1,i} - \varepsilon\phi_{2j,i}) \end{aligned} \tag{12a, b}$$

where the effect of eccentricity is represented by the parameter ϵ and the effect of torsion indirectly by the vibration shapes.

Let i denote the vibration mode of the system that has the largest participation factor, which is usually the first flexural mode, and assume the equipment to be tuned to this mode, i.e., $\omega_e = \omega_i$. By neglecting the effects of other untuned modes, one may write

$$\{u\}_{\mathrm{s}}^* \cong Y_i\{\phi\}_i^* \tag{13}$$

Then, the displacement u_f of the attachment point of the equipment becomes

$$u_f \cong Y_i \tag{14}$$

Substituting equations (13) and (14) into equations (9a,b), premultiplying equation (9a) by $(\{\phi\}_i^*)^{\mathrm{T}}$, and utilizing the properties of modal orthogonality, one can show that

$$\begin{bmatrix} M_i^* & 0 \\ 0 & m_e \end{bmatrix} \begin{Bmatrix} \ddot{Y}_i \\ \ddot{u}_e \end{Bmatrix} + \begin{bmatrix} M_i^*\omega_i^2 + k_e & -k_e \\ -k_e & k_e \end{bmatrix} \begin{Bmatrix} Y_i \\ u_e \end{Bmatrix} = \begin{Bmatrix} M_i^*\Gamma_i^* \\ m_e \end{Bmatrix} \ddot{u}_{\mathrm{g}} \tag{15}$$

For the case where the effective mass ratio of equipment is small, i.e. $\gamma_i = m_e/M_i^* \ll 1$ and $k_e = m_e\omega_e^2 = m_e\omega_i^2$, the frequencies for the reduced system (15) can be solved as

$$\hat{\omega}_1 = \left(\sqrt{1+\frac{\gamma_i}{4}} - \frac{\sqrt{\gamma_i}}{2}\right)\omega_i, \quad \hat{\omega}_2 = \left(\sqrt{1+\frac{\gamma_i}{4}} + \frac{\sqrt{\gamma_i}}{2}\right)\omega_i \tag{16}$$

and the associated shapes of vibration as

$$\begin{Bmatrix} \hat{Y}_i \\ \hat{u}_e \end{Bmatrix}_1 = \begin{Bmatrix} \dfrac{-\gamma_i + \sqrt{\gamma_i^2 + 4\gamma_i}}{2} \\ 1 \end{Bmatrix}, \quad \begin{Bmatrix} \hat{Y}_i \\ \hat{u}_e \end{Bmatrix}_2 = \begin{Bmatrix} \dfrac{-\gamma_i - \sqrt{\gamma_i^2 + 4\gamma_i}}{2} \\ 1 \end{Bmatrix}, \tag{17}$$

Correspondingly, the participation factors are

$$\begin{aligned} \hat{\Gamma}_1 &= \frac{\gamma_i + 4 + \sqrt{\gamma_i^2 + 4\gamma_i}}{2(\gamma_i + 4)} + \frac{\sqrt{\gamma_i^2 + 4\gamma_i}}{\gamma_i(\gamma_i + 4)}\Gamma_i^* \\ \hat{\Gamma}_2 &= \frac{\gamma_i + 4 - \sqrt{\gamma_i^2 + 4\gamma_i}}{2(\gamma_i + 4)} - \frac{\sqrt{\gamma_i^2 + 4\gamma_i}}{\gamma_i(\gamma_i + 4)}\Gamma_i^* \end{aligned} \tag{18}$$

Neglecting the higher-order terms of γ_i, the preceding expressions reduce to

$$\hat{\Gamma}_1 \cong \frac{1}{2} + \frac{\Gamma_i^*}{2\sqrt{\gamma_i}}, \qquad \hat{\Gamma}_2 \cong \frac{1}{2} - \frac{\Gamma_i^*}{2\sqrt{\gamma_i}} \tag{19}$$

As a consequence, the modal accelerations for the equipment are

$$(a_e)_1 \cong \frac{\Gamma_i^*}{2\sqrt{\gamma_i}}\left(1 + \frac{\sqrt{\gamma_i}}{\Gamma_i^*}\right)\hat{S}a_1, \quad (a_e)_2 \cong -\frac{\Gamma_i^*}{2\sqrt{\gamma_i}}\left(1 - \frac{\sqrt{\gamma_i}}{\Gamma_i^*}\right)\hat{S}a_2 \tag{20}$$

where $\hat{S}a_i = Sa(\hat{\omega}_i, \hat{\xi}_i)$, $\hat{\xi}_i$ denotes the damping ratio of the ith mode of vibration of the equipment–structure system, and $Sa(\hat{\omega}_i, \hat{\xi}_i)$ the spectral acceleration of a single-DOF system that has a damping ratio of $\hat{\xi}_i$ and a frequency of $\hat{\omega}_i$ subjected to the excitation of the earthquake considered. By the CQC method,[8] the maximum acceleration for the equipment can be computed as

$$\hat{a}_{e,\max} \cong \frac{\Gamma_i^*}{2\sqrt{\gamma_i}}\left[(\hat{S}a_1)^2 + (\hat{S}a_2)^2 - 2\hat{\rho}_{12}\hat{S}a_1\hat{S}a_2\right]^{1/2} \tag{21}$$

where the coefficient of correlation is

$$\hat{\rho}_{12} \cong \frac{2(\hat{\xi}_1 + \hat{\xi}_2)(\hat{\xi}_1\hat{\xi}_2)^{1/2}}{\gamma_i + (\hat{\xi}_1 + \hat{\xi}_2)^2} \tag{22}$$

Equation (21) indicates that the order of magnitude for the maximum acceleration of the equipment is $\gamma_i^{-1/2}$. Since the effective mass ratio γ_i is small for the equipment–structure system considered, it is expected that the maximum acceleration experienced by an equipment item tuned to the fixed-base structure can be quite large in practice.

Response of equipment mounted in torsional building with base isolation

According to Reference 9, the effect of torsion can be minimized if the CR of the isolators coincides with the CM of the base-isolated structure, i.e.

$$\sum m_i d_i + m_b d_b = 0 \tag{23}$$

Based on this condition, the equations of motion as given in equation (3) for the base-isolated torsional building can be rewritten,

$$[\bar{m}]\{\ddot{u}\} + [k]\{u\} = -[\bar{m}]\{l_x\}\ddot{u}_g \tag{24}$$

where the quantities $[k]$, $\{u\}$, and $\{l_x\}$ are identical to those given in equations 4(b)–(d), and the mass matrix $[\bar{m}\}$ is

$$[\bar{m}] = \begin{bmatrix} m_t & 0 & ([m]_s\{l_x\}_s)^T \\ 0 & m_t r_t^2 & ([m]_s\{l_{\theta_s}\})^T \\ ([m]_s\{l_x\}_s)^T & ([m]_s\{l_\theta\}_s)^T & [m]_s \end{bmatrix} \tag{25}$$

Again, consider the case where the equipment is attached to point f of the jth floor of the torsional building with eccentricity ϵ from the CM. By transforming the displacement and rotation of the jth floor from the CM to the attachment point f, i.e. using equation (8), the equations of motion (24) can be rewritten,

$$[\bar{m}]^*\{\ddot{u}\}^* + [k]^*\{u\}^* = -[\bar{m}]^*\{l_x\}^*\ddot{u}_g \tag{26}$$

where

$$[\bar{m}]^* = \begin{bmatrix} m_t & 0 & ([m]_s^*\{l_x\}_s^*)^T \\ 0 & m_t r_t^2 & ([m]_s^*\{l_\theta\}_s^*)^T \\ ([m]_s^*\{l_x\}_s^*)^T & ([m]_s^*\{l_\theta\}_s^*)^T & [m]_s^* \end{bmatrix}$$

$$[k]^* = \begin{bmatrix} k_{bx} & 0 & 0 \\ 0 & k_{b\theta} & 0 \\ 0 & 0 & [k]_s^* \end{bmatrix} \quad \{u\}^* = \begin{Bmatrix} u_b \\ \theta_b \\ [u]_s^* \end{Bmatrix} \quad \{l_x\}^* = \begin{Bmatrix} 1 \\ 0 \\ \{0\} \end{Bmatrix} \tag{27a–d}$$

Here, the quantities for the transformed system are denoted with superscript '*'.

To solve for the vibration mode of the base-isolated building, to which the equipment is tuned, corresponding to the ith mode of the fixed-base structure that has the largest participation factor, let us adopt the same approximation as the one given in equation (13), i.e., $\{u\}_s^* \cong Y_i\{\phi\}_i^*$, where $\{\phi\}_i^*$ denotes the ith mode of vibration of the fixed-base structure after the transformation (8). Substituting this expression into equation (26) and making use of the modal orthogonality property, one obtains

$$\begin{bmatrix} m_t & 0 & M_i^*\Gamma_{xi}^* \\ 0 & m_t r_t^2 & M_i^*\Gamma_{\theta i}^* \\ M_i^*\Gamma_{xi}^* & M_i^*\Gamma_{\theta i}^* & M_i^* \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{\theta} \\ \ddot{Y}_i \end{Bmatrix} + \begin{bmatrix} k_{bx} & 0 & 0 \\ 0 & k_{b\theta} & 0 \\ 0 & 0 & M_i^*\omega_i^2 \end{bmatrix}\begin{Bmatrix} u \\ \theta \\ Y_i \end{Bmatrix} = -\begin{bmatrix} m_t & 0 & M_i^*\Gamma_{xi}^* \\ 0 & m_t r_t^2 & M_i^*\Gamma_{\theta i}^* \\ M_i^*\Gamma_{xi}^* & M_i^*\Gamma_{\theta i}^* & M_i^* \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}\ddot{u}_g \tag{28}$$

in which M_i^* and Γ_{xi}^* have been given in equations (12a,b), and the participation factor $\Gamma_{\theta i}^*$ for the torsional response is

$$\Gamma_{\theta i}^* = (\{\phi\}_i^{*T}[m]_s^*\{l_\theta\}_s^*/M_i^* = \Gamma_{\theta i}(\phi_{2j-1,i} - \varepsilon\phi_{2j,i}) \tag{29}$$

In general, the frequencies ω_{bx} and $\omega_{b\theta}$ of the base isolators are much smaller than that of the ith mode (i.e., the predominant mode) of the fixed-based structure. Thus,

$$\omega_{bx}^2 = \frac{k_{bx}}{m_i} \ll \omega_i^2, \quad \omega_{b\theta}^2 = \frac{k_{b\theta}}{m_t r_t^2} \ll \omega_i^2, \quad \sigma_i = \frac{\omega_{bx}^2}{\omega_i^2} \ll 1, \quad \eta = \frac{\omega_{b\theta}^2}{\omega_{bx}^2} \tag{30}$$

Further, it is assumed that the torsional frequency of the base isolators is larger than the flexural frequency, i.e. $\omega_{bx} < \omega_{b\theta}$ or $\eta > 1$. By the definitions given in equation (30) and neglecting terms of higher orders in σ_i, one can solve from equation (28) the following:

$$\hat{\omega}_1 \cong \omega_{bx}, \quad \hat{\omega}_2 \cong \omega_{b\theta}, \quad \hat{\omega}_3 \cong \frac{\omega_i}{\sqrt{1 - P_{xi} - P_{\theta i}}} \tag{31a–c}$$

where

$$P_{xi} = M_i^* \Gamma_{xi}^{*2} / m_t, \quad P_{\theta i} = M_i^* \Gamma_{\theta i}^{*2} / (m_t r_t^2) \tag{32}$$

Correspondingly, the modal vectors are

$$\begin{Bmatrix} \hat{u}_b \\ \hat{\theta}_b \\ \hat{Y}_i \end{Bmatrix}_1 \cong \begin{Bmatrix} 1 + \dfrac{P_{\theta i}\sigma_i}{1-\eta} \\ -\dfrac{\Gamma_{xi}}{\Gamma_{\theta i}} \dfrac{\sigma_i}{1-\eta} P_{\theta i} \\ \Gamma_{xi}\sigma_i \end{Bmatrix}, \quad \begin{Bmatrix} \hat{u}_b \\ \hat{\theta}_b \\ \hat{Y}_i \end{Bmatrix}_2 \cong \begin{Bmatrix} \dfrac{\Gamma_{\theta i}}{\Gamma_{xi}} \dfrac{\eta^2 \sigma_i}{1-\eta} P_{xi} \\ 1 - \dfrac{\eta^2\sigma_i}{1-\eta} P_{xi} \\ \Gamma_{\theta i}\eta\sigma_i \end{Bmatrix},$$

$$\begin{Bmatrix} \hat{u}_b \\ \hat{\theta}_b \\ \hat{Y}_i \end{Bmatrix}_3 \cong \begin{Bmatrix} -\dfrac{P_{xi}}{\Gamma_{xi}} [1 + (1 - P_{xi} - P_{\theta i})\sigma_i] \\ -\dfrac{P_{\theta i}}{\Gamma_{\theta i}} [1 + (1 - P_{xi} - P_{\theta i})\eta\sigma_i] \\ 1 \end{Bmatrix} \tag{33a–c}$$

the effective masses are

$$\begin{aligned} \hat{M}_1 &\cong m_t \left(1 + \frac{P_{\theta i}\sigma_i}{1-\eta}\right)^2 + \frac{\Gamma_{xi}^{*2}}{\Gamma_{\theta i}^{*2}} \frac{\eta\sigma_i^2}{(1-\eta)^2} P_{\theta i}^2 m_t r_t^2 + M_i^* \Gamma_{xi}^{*2} \sigma_i \\ \hat{M}_2 &\cong m_t r_t^2 \left(1 - \frac{P_{xi}\eta^2\sigma_i}{1-\eta}\right)^2 + \frac{\Gamma_{\theta i}^{*2}}{\Gamma_{xi}^{*2}} \frac{\eta^3\sigma_i^2}{(1-\eta)^2} P_{xi}^2 m_t + M_i^* \Gamma_{\theta i}^{*2} \eta\sigma_i \\ \hat{M}_3 &\cong (1 - P_{xi} - P_{\theta i}) M_i^* \end{aligned} \tag{34a–c}$$

and the modal participation factors are

$$\hat{\Gamma}_1 \cong \frac{m_t}{\hat{M}_1}\left(1 + \frac{P_{\theta i}\sigma_i}{1-\eta}\right), \quad \hat{\Gamma}_2 \cong \left(\frac{\Gamma_\theta}{\Gamma_{xi}} \frac{\eta^2\sigma_i}{1-\eta} m_t P_{xi} + M_i^* \Gamma_{xi}^* \Gamma_{\theta i}^* \eta\sigma_i\right) \Big/ \hat{M}_2$$
$$\hat{\Gamma}_3 \cong \Gamma_{xi}\sigma_i \tag{35a–c}$$

By the relations (5) and (8), the displacements of the superstructure with respect to the ground can be expressed as

$$\{u\}_{sg}^* = \{u\}_s^* + \{l_x\}_s^* u_b + \{l_\theta\}_s^* \theta_b \tag{36}$$

or equivalently

$$\begin{Bmatrix} u_b \\ \theta_b \\ \{u\}_{sg}^* \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \{l_x\}_s^* & \{l_\theta\}_s^* & \{1\} \end{bmatrix} \begin{Bmatrix} u_b \\ \theta_b \\ \{u\}_s^* \end{Bmatrix} \tag{37}$$

where {1} denotes a column with all its entries set equal to unity. With this relation, the displacements of the superstructure with respect to the foundation solved from equation (26) can be transformed to those with respect to the ground. Since the frequencies are not affected by transformation and since the relation (37) is obeyed by the modal vectors, the effective masses and modal participation factors remain unchanged. Thus, the procedure described in Reference 7 for solving the response of equipment items mounted on torsional buildings can be adopted herein.

As was stated, the equipment is attached to the jth floor of the base-isolated building with an eccentricity of ϵ from the CM. Based on the transformations of equations (36) and (8) and the definitions for effective masses and modal participation factors, and by letting the displacement of the equipment DOF in the modal vectors with reference to the ground equal one, one can express the effective mass ratio γ_k^* and modal participation factor $\hat{\Gamma}_k^*$ as

$$\gamma_k^* = \frac{m_e(\hat{Y}_{i,k} + \hat{u}_{b,k} + d_f\hat{\theta}_{b,k})^2}{\hat{M}_k}, \qquad \hat{\Gamma}_k^* = \hat{\Gamma}_k(\hat{Y}_{i,k} + \hat{u}_{b,k} + d_f\hat{\theta}_{b,k}) \tag{38a, b}$$

where $k = 1, 2, 3$, and d_f denotes the distance from the point of attachment of the equipment to the CR of the foundation.

For an equipment item having its frequency ω_e higher than the isolator frequencies ω_{bx} and $\omega_{b\theta}$, its response will reach the maximum when the frequency is tuned to the first flexural mode of the superstructure, i.e.,

$$\omega_e = \frac{\omega_i}{\sqrt{1 - P_{xi} - P_{\theta i}}} \tag{39}$$

In this study, only the tuned mode and the first two (i.e., rigid body) modes of the base-isolated structure will be considered. According to Reference 7, the maximum accelerations of the equipment associated with the two detuned modes are

$$(a_e)_1 \cong \frac{\omega_e^2}{\omega_e^2 - \omega_{bx}^2}\,\hat{\Gamma}_1^* Sa(\omega_{bx}, \hat{\xi}_1), \quad (a_e)_2 \cong \frac{\omega_e^2}{\omega_e^2 - \omega_{b\theta}^2}\,\hat{\Gamma}_2^* Sa(\omega_{b\theta}, \hat{\xi}_2) \tag{40a, b}$$

where $\hat{\xi}_1$ and $\hat{\xi}_2$ denote the damping ratios of the first two modes of the base-isolated system. By substituting equation (33a) into (38b) and then into equation (40a), and assuming the frequency ratio $\eta > 1$, the equipment acceleration caused by the first mode can be derived,

$$(a_e)_1 = Sa(\omega_{bx}, \hat{\xi}_1) + O(\sigma_i) \tag{41}$$

where $O(\cdot)$ denotes the order of magnitude. Similarly, the equipment acceleration caused by the second mode can be derived as

$$(a_e)_2 = 0 + O(\sigma_i) \tag{42}$$

As was stated previously, the equipment is tuned to the first flexural frequency of the superstructure. It follows that this frequency will be split into two frequencies, which can be computed using equation (45) of Reference 7, along with the substitution of equations (33c) and (38a), as

$$\bar{\omega}_{i,1} = (-\delta + \sqrt{1 + \delta^2})\hat{\omega}_3, \qquad \bar{\omega}_{i,2} = (\delta + \sqrt{1 + \delta^2})\hat{\omega}_3 \tag{43}$$

in which $\hat{\omega}_3$ has been defined in equation (31c) and δ is

$$\delta = \left(1 - \frac{P_{xi}}{\Gamma_{xi}} - d_f\frac{P_{\theta i}}{\Gamma_{\theta i}}\right)\frac{1}{4\sqrt{1 - P_{xi} - P_{\theta i}}} \tag{44}$$

Further, for the case of light equipment, the spectral accelerations associated with these two split modes can be calculated from equation (46) of Reference 7 as

$$(a_e)_{i,1} \cong \left[\frac{1}{2}+\frac{1}{2}\hat{\Gamma}_3\left(\frac{\hat{M}_3}{m_e}\right)^{1/2}\right] Sa(\hat{\omega}_3,\hat{\xi}_3), \quad (a_e)_{i,2} \cong \left[\frac{1}{2}-\frac{1}{2}\hat{\Gamma}_3\left(\frac{\hat{M}_3}{m_e}\right)^{1/2}\right] Sa(\hat{\omega}_3,\hat{\xi}_3), \tag{45}$$

By the substitution of equations (34c) and (35c), the following can be derived:

$$\begin{aligned}(a_e)_{i,1} &\cong \left(\frac{1}{2}+\frac{\sigma_i}{2\sqrt{\gamma_i}}\Gamma^*_{xi}\sqrt{1-P_{xi}-P_{\theta i}}\right) Sa(\hat{\omega}_3,\hat{\xi}_3)\\ (a_e)_{i,2} &\cong \left(\frac{1}{2}-\frac{\sigma_i}{2\sqrt{\gamma_i}}\Gamma^*_{xi}\sqrt{1-P_{xi}-P_{\theta i}}\right) Sa(\hat{\omega}_3,\hat{\xi}_3)\end{aligned} \tag{46a, b}$$

For the case where the order of magnitude for σ_i is the same as or smaller than that of $\sqrt{\gamma_i}$, one can observe from the preceding two expressions that the order of magnitude of the accelerations associated with the tuned modes is 1. By the CQC method, the maximum acceleration of the equipment can be computed as a superposition of the four expressions (41), (42), (46a) and (46b):

$$\begin{aligned}\hat{a}_{e,\max} \cong \Bigg\{ & Sa^2(\omega_{bx},\hat{\xi}_1) + \left[\frac{1}{2}+\frac{\sigma_i^2}{2\gamma_i}\Gamma^{*2}_{xi}(1-P_{xi}-P_{\theta i})\right] Sa^2(\hat{\omega}_3,\hat{\xi}_3)\\ & + \hat{\rho}_{i1,i2}\left[\frac{1}{2}-\frac{\sigma_i^2}{2\gamma_i}\Gamma^{*2}_{xi}(1-P_{xi}-P_{\theta i})\right] Sa^2(\hat{\omega}_3,\hat{\xi}_3)\Bigg\}^{1/2}\end{aligned} \tag{47}$$

where

$$\hat{\rho}_{i1,i2} \cong \frac{2(\hat{\xi}_3+\hat{\xi}_3)(\hat{\xi}_3\hat{\xi}_3)^{1/2}}{4\delta^2+(\hat{\xi}_3+\hat{\xi}_3)^2} = \frac{\hat{\xi}_3^2}{\delta^2+\hat{\xi}_3^2} \tag{48}$$

For the case where σ_i has an order of magnitude same as or smaller than $\sqrt{\gamma_i}$, it can be seen from equation (47) that the order of magnitude for the acceleration of an equipment mounted on a base-isolated torsional building is 1. In comparison, the order of magnitude for the acceleration of the equipment mounted on a building with fixed base is $\gamma_i^{-1/2}$, as indicated by equation (21). Realizing that $\gamma_i\ (= m_e/M_i^*) \ll 1$ for light equipment, one can therefore confirm the effectiveness of the base isolators in reducing the response of the equipment mounted to a torsional building even under the tuning condition.

It should be noted that for a specific base-isolated system subjected to a specific earthquake (i.e. with a given spectrum), the modal accelerations of the equipment given in equations (46) are determined primarily by the modal participation factor Γ_i^* and the effective mass ratio γ_i, both of which are functions of the factor $(\phi_{2j-1,i}-\epsilon\phi_{2j,i})^2$. Such a factor represents, in essence, a combination of the effect of eccentricity of the attached equipment, through the parameter ϵ, and the effect of torsion of the primary structure, through the modal vector $\{\phi\}_i$.[7] From equation (12a), one observes that the effective mass M_i^* decreases as the factor $(\phi_{2j-1,i}-\epsilon\phi_{2j,i})^2$ increases, which in turn results in the increase of the effective mass ratio γ_i, according to the definition $\gamma_i = m_e/M_i^*$, and the interaction between the equipment and structure. An overall result is the reduction in response of the equipment. However, from equation (12b), one also observes that an increase in $(\phi_{2j-1,i}-\epsilon\phi_{2j,i})^2$ may also cause the participation factor Γ_i^* to increase, which, on the other hand, tends to enlarge the equipment response. In the numerical studies to follow, it will be demonstrated that whenever the factor $(\phi_{2j-1,i}-\epsilon\phi_{2j,i})^2$ equals zero, the equipment response reaches its minimum. This implies that the optimal point for mounting the equipment is the point which remains undisplaced during vibration of the tuned mode.

Table I. Dynamic characteristics of base-isolated shear building

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
ω_i (rps)	1·6833	1·8927	11·0613	12·0288	18·3425	21·5198
$\{\phi\}_i$	0·9324	– 0·0015	– 0·8731	0·0411	– 0·5517	0·0131
	0·0002	0·9411	– 0·0082	– 0·6975	– 0·0080	– 0·3062
	0·9764	– 0·0011	– 0·0197	0·0020	1·0000	– 0·0248
	0·0003	0·9801	0·0021	0·1963	0·0065	1·0000
	1·0000	– 0·0010	1·0000	– 0·0488	– 0·5543	0·0144
	0·0003	1·0000	0·0118	1·0000	– 0·0040	– 0·06359
$M_i(10^4\,\text{kg})$	15·94	76·27	9·58	39·19	9·36	35·61
Γ_{xi}	1·0320	– 0·0003	– 0·0372	0·0004	– 0·0088	0·0000
$\Gamma_{\theta i}$	0·0013	1·0330	– 0·0021	– 0·0369	– 0·0002	– 0·0056
ξ_i	0·1318	0·1175	0·0300	0·0293	0·0288	0·0300

NUMERICAL APPLICATIONS

The response spectrum that will be adopted in this study is the smooth design spectrum proposed by Newmark and Hall[10] for the 1940 El Centro earthquake (N–S component), assuming 0·1 g maximum ground acceleration, 3 per cent damping ratio, and a cumulative probability of 50 per cent. For convenience, it is assumed that the installation of a light equipment does not alter the damping property of the primary structure, which is assumed to be of the Rayleigh type.[7] The solutions obtained by the present approach will be compared with a *reference* solution obtained by solving directly the eigenvalue problem for the entire equipment–structure system using the modal synthesis procedure.

Example 1. Equipment mounted in a base-isolated shear building

Consider a two-storey shear building with a very small eccentricity, for which the following properties are assumed: mass $m_1 = 60\,000$ kg, $m_2 = 50\,000$ kg, radius of gyration $r_1 = r_2 = 2{\cdot}0$ m, flexural stiffness $k_{x1} = 7 \times 10^6$ N/m, $k_{x2} = 6 \times 10^6$ N/m, torsional stiffness $k_{\theta 1} = 40 \times 10^6$ N-m/rad, $k_{\theta 2} = 36 \times 10^6$ N-m/rad, and floor eccentricity $e_{11} = e_{12} = e_{22} = 0.01$ m. The following properties are assumed for the base with isolators: $m_b = 60\,000$ kg, radius of gyration $r_b = 2{\cdot}5$ m, flexural stiffness $k_{bx} = 0{\cdot}5 \times 10^6$ N/m, torsional stiffness $k_{b\theta} = 5 \times 10^6$ N-m/rad. In addition, the CM and CR of the base isolators coincide with the CM of the superstructure. Based on the assumption of Rayleigh damping, the damping ratio for each mode is determined by setting the damping ratios for the third and sixth modes (i.e. the first and fourth modes of the superstructure) equal to 0·03. By so doing, much larger damping ratios can be computed, and thereby assumed, for the first two modes, as is typical with base isolators. All the dynamic properties computed for the primary base-isolated system have been listed in Table I. As can be seen, the first two modes represent basically the rigid-body modes, and their frequencies are much lower than the other vibration modes of the superstructure. Nevertheless, the participation factors of the first two modes are much larger than the other ones, say, by a factor larger than 25, indicating that the structural behaviour of system is dominated primarily by the two rigid-body modes. The great reduction in the participation factors of the vibration modes of the superstructure, compared with the fixed-base case given in Table II of Reference 7, can be attributed to the installation of base isolators. Because of this, no significant amplification will be induced by the equipment, even when it is tuned to the vibration modes of the superstructure.

Assuming an equipment item of mass $m_e = 100$ kg and $\xi_e = 0{\cdot}03$ to be attached to the CM of the second storey of the building, i.e., with $\epsilon = 0$ m, one can compute the modal acceleration of the equipment as a function of the equipment frequency ω_e. From the results plotted in Figure 3, one observes that the tuned modes always appear as a pair, which are roughly of equal magnitudes but opposite in sign, and that the tuning effect for the flexural modes (with $\omega_3 = 11{\cdot}061$ rps and $\omega_5 = 18{\cdot}343$ rps) is much larger than that for

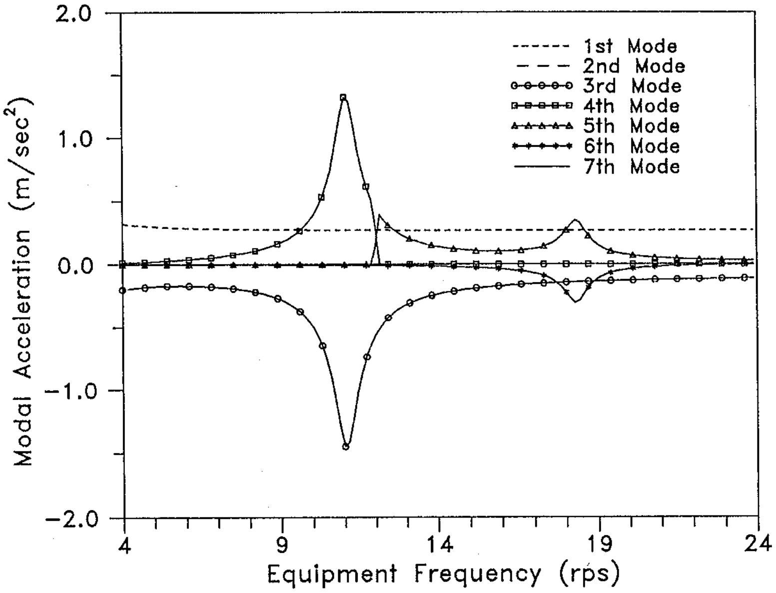

Figure 3. Modal acceleration of equipment in base-isolated shear building

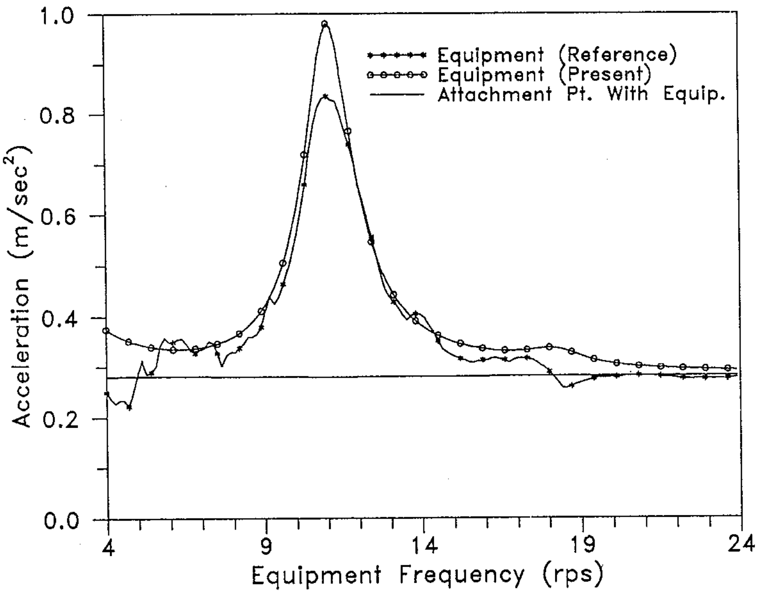

Figure 4. Peak acceleration of equipment in base-isolated shear building

the torsional modes (with $\omega_4 = 12{\cdot}029$ rps and $\omega_6 = 21{\cdot}520$ rps). The modal responses have been superimposed using the CQC procedure to yield the peak response for the equipment in Figure 4, in which the structure response at the attachment point of the equipment is also shown. As can be seen, the equipment response is dominated primarily by the flexural modes, and the result obtained by the present approximate

Table II. Dynamic characteristics of base-isolated torsional building

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
ω_i (rps)	1.6828	1·8928	10·7013	12·4337	17·9128	22·0422
$\{\phi\}_i$	0·9313	− 0·0440	− 0·8820	− 0·8200	0·55880	0·3312
	0·0057	0·9413	− 0·1700	0·6828	− 0·0440	− 0·3080
	0·9760	− 0·0340	− 0·0150	− 0·0570	1·0000	− 0·6320
	0·0083	0·9800	0·0419	− 0·2020	0·1639	1·0000
	1·0000	− 0·0290	1·0000	1·0000	− 0·5470	0·3680
	0·0097	1·0000	0·2463	− 0·9720	− 0·1160	− 0·6320
$M_i(10^4$ kg)	15·92	76·30	12·01	46·41	10·36	39·28
Γ_{xi}	1·0330	− 0·0081	− 0·0321	− 0·0057	− 0·0084	0·0009
$\Gamma_{\theta i}$	0·0382	1·0330	− 0·0371	0·0286	− 0·0040	− 0·0048
ξ_i	0·1300	0·1159	0·0300	0·0288	0·0285	0·0300

procedure appears to be in consistence with that based on solution of the complete eigenvalue solution (indicated as reference solution in the figure). A comparison of the solutions given in Figures 3 and 4 with those of References 7 and 11 for the fixed-base case using similar data indicates that the responses of both the equipment and the primary structure have been greatly reduced, say, by a factor larger than 20, although the effect of tuning remains observable.

Example 2. Equipment mounted in a base-isolated torsional building

All the data assumed for the two-storey torsional building with base isolation are the same as those adopted in Example 1, except that eccentricities of larger values are used: $e_{11} = e_{12} = e_{22} = 0{\cdot}30$ m, and that the torsional stiffness of the base isolators is slightly increased to $k_{b\theta} = 7 \times 10^6$ N-m/rad. The dynamic properties solved for this system are listed in Table II. As can be seen, the first two modes are rigid body modes, and their participation factors are much larger than the other ones, say, by a factor larger than 25. Again, the great reduction in the participation factors of vibration modes of the superstructure can be attributed to the installation of base isolators, compared with Table I of Reference 7 for the fixed-base case. Because of this, it can be expected that the behaviour of the equipment–structure system is dominated primarily by the rigid body modes, and that the tuning effect of the equipment will be drastically reduced.

Again, let us assume that an equipment item of mass $m_e = 100$ kg, $\xi_e = 0{\cdot}03$ and $\epsilon = 0$ m is attached to the CM of the second storey of the torsional building. From the modal acceleration of the equipment plotted in Figure 5, one observes that the effect of tuning is significant whenever the equipment frequency coincides with the frequencies of the first few modes of the superstructure. The synthesized response of the equipment computed using the present approach has been shown to be generally consistent with that obtained by solving the complete eigenvalue solution in Figure 6, where the structure response at the attachment point of the equipment is also plotted.

Example 3. Effect of eccentricity of equipment

Exactly the same data as those used in Example 2 are used in this example. The equipment with a mass of $m_e = 100$ kg is attached to the second storey of the building. The first case to be considered is one with the equipment tuned to the third frequency of the base-isolated torsional building, i.e. the first frequency of the superstructure. The eccentricity ϵ of the equipment from the CM of the second storey is allowed to vary from − 10 to 10 m. The results computed for the modal acceleration and peak acceleration of the equipment (along with that of the attachment point) have been plotted in Figures 7 and 8, respectively. From the dynamic properties given in Table II, the modal components of the attachment point of the equipment

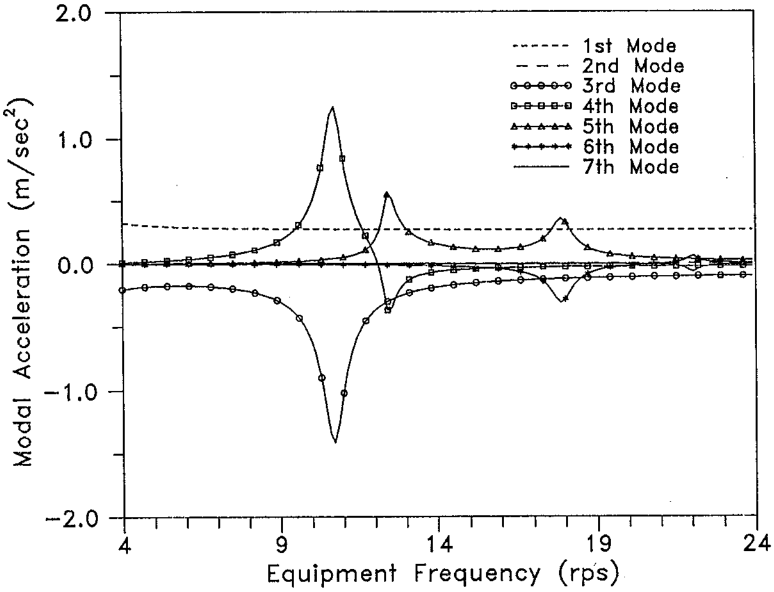

Figure 5. Modal acceleration of equipment in base-isolated torsional building

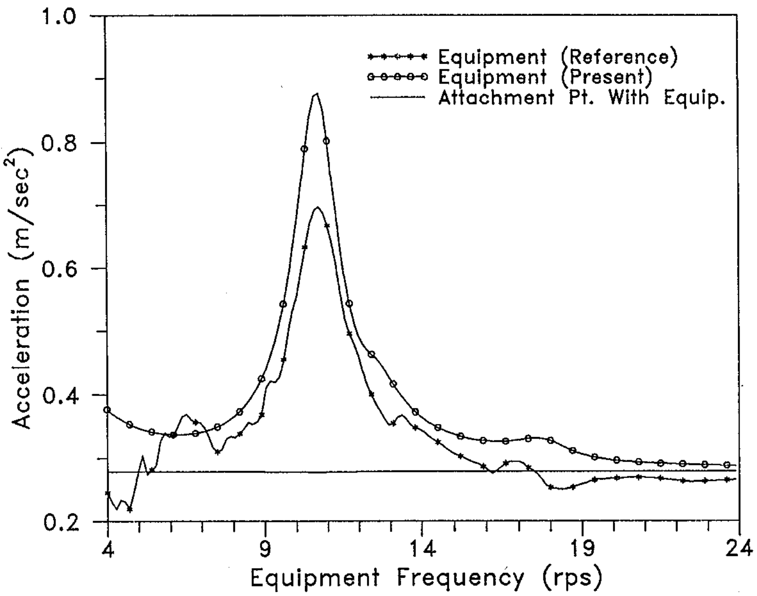

Figure 6. Peak acceleration of equipment in base-isolated torsional building

associated with the third (tuned) mode can be obtained as: $\phi_{5,3} = 1{\cdot}0$ and $\phi_{6,3} = 0{\cdot}2463$. By setting the factor $(\phi_{5,3} - \epsilon\phi_{6,3})^2$ equal to zero, it can be solved $\epsilon = 4{\cdot}06$ m. This is the point where the equipment can remain stationary during vibration of the tuning mode. As can be seen from Figure 8, the equipment response reaches its minimum when it is located near the stationary point, i.e., with $\epsilon = 4{\cdot}06$ m. The result obtained by

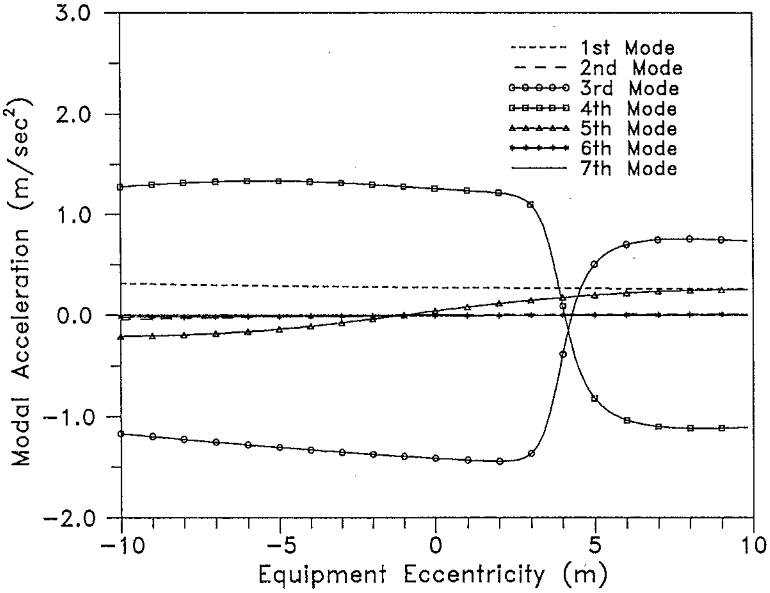

Figure 7. Modal acceleration of equipment vs. eccentricity (case 1)

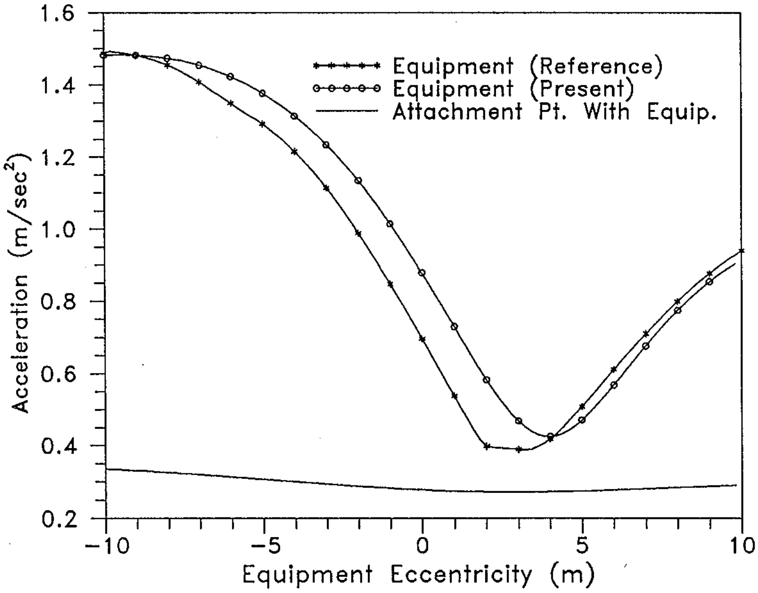

Figure 8. Peak acceleration of equipment vs. eccentricity (case 1)

solving the complete eigenvalue problem, i.e. the reference solution, shows slight deviation from the present one, in which only the rigid body and tuned modes are considered.

The second case to be considered here is that the equipment is tuned instead to the fourth mode of vibration. The results have been plotted in Figures 9 and 10. For this case, the modal components of the

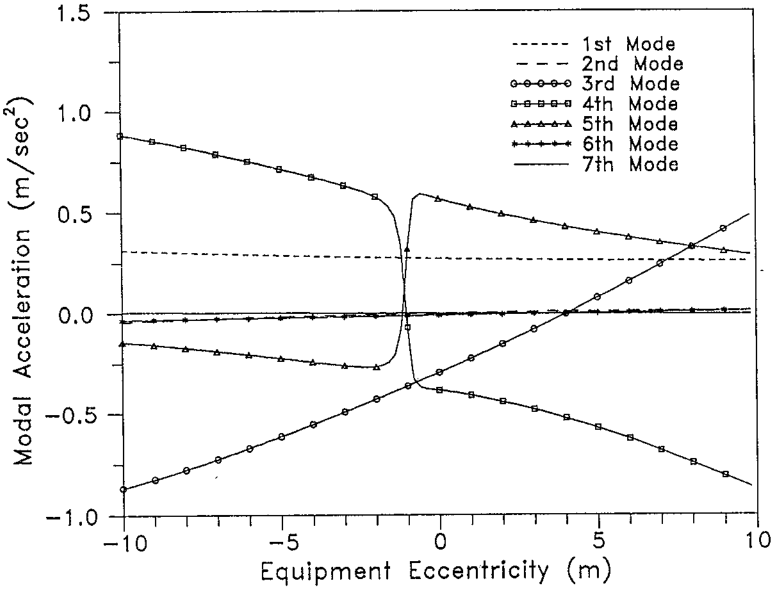

Figure 9. Modal acceleration of equipment vs. eccentricity (case 2)

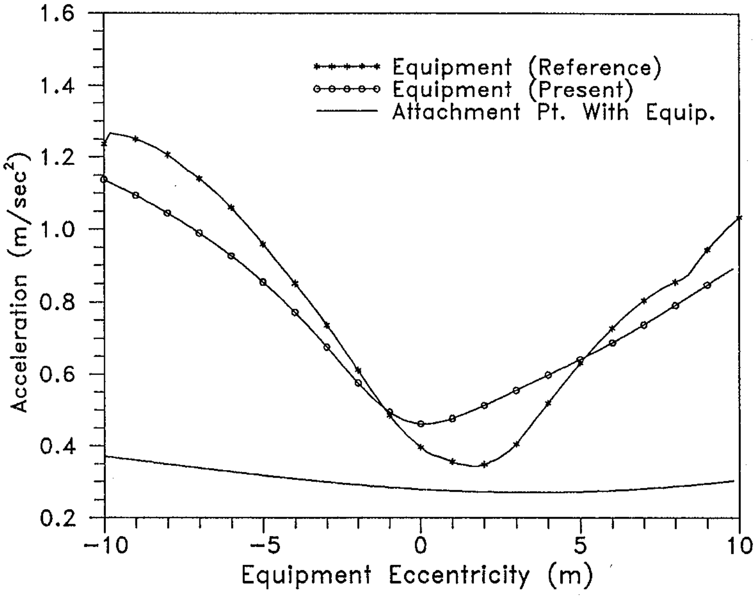

Figure 10. Peak acceleration of equipment vs. eccentricity (case 2)

attachment point of the equipment associated with the fourth (tuned) mode can be obtained from Table II as $\phi_{5,4} = 1{\cdot}0$ and $\phi_{6,4} = -0{\cdot}972$. By setting the factor $(\phi_{5,4} - \epsilon\phi_{6,4})^2$ equal to zero, it can be solved $\epsilon = -1{\cdot}02$ m. It can be seen from Figure 9 that the modal acceleration reaches zero at this point. The minimum response of the equipment has shifted slightly away from this point due to the effect of higher modes (see Figure 10).

CONCLUDING REMARKS

Based on the results presented in this paper, several conclusions can be made for equipment items mounted on torsional buildings of which the centre of rigidity of the base isolators coincide with the centre of mass of the entire system. First, the results computed by the present perturbation approach are generally consistent with those based on the complete eigenvalue solution. Second, the response of the equipment and the torsional building to which it is mounted, can be effectively reduced through installation of the base isolators. Third, the dynamic characteristics of the equipment mounted in a base-isolated torsional building are similar to those for the fixed-base case; the only difference is that the equipment response is drastically reduced due to the effect of isolation. Finally, there exists an optimal point for mounting the equipment, which turns out to be the stationary point of each floor of the torsional building under the vibration of the tuning mode, as determined by the condition $(\phi_{2j-1,i} - \epsilon\phi_{2j,i})^2 = 0$.

ACKNOWLEDGEMENTS

The research is sponsored partly by the National Science Council of the Republic of China through its Grant Nos. NSC83-0416-E002-018 and NSC84-2221-E002-049. The writers would like to thank the reviewers for their valuable comments.

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